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CAN AGGREGATE DIRECT TRAVEL DEMAND MODELS WORK?

by

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ABSTRACT

Applications of aggregate spatial direct demand models to interzonal urban transportation are not frequently found in the literature. This is no doubt due to the difficulties encountered in calibrating such models. They are therefore seldom used by researchers. The purpose of this paper is to suggest that adequate parameter interpretation of the network variables and appropriate functional form selection may produce satisfactory results. Our proposition is illustrated by an application to the demand for interzonal work trips, by car and public transit, in Metropolitan Montreal. Moreover, comparisons of results obtained in the usual way by neglecting the null observations of the origin-destination matrix indicates that these are much less efficient than results obtained by taking them into account.

KEY-WORDS : AGGREGATE DATA, DIRECT TRAVEL DEMAND, LEAST-SQUARES, TOBIT, MONTREAL

RÉSUMÉ

On trouve peu d'applications des modèles agrégés de demande directe spatiale au transport urbain interzonal. Cela provient sans doute du fait que ces modèles semblent être difficiles à calibrer. Pour cette raison, les chercheurs utilisent rarement ce genre de modèles. Le but de cet article est de suggérer qu'une interprétation adéquate des paramètres des variables de réseaux et une sélection attentive de la forme fonctionnelle appropriée peuvent effectivement conduire à des résultats satisfaisants. Notre propos est illustré à l'aide d'une application à la demande de transport interzonal, pour le motif travail, dans la région métropolitaine de Montréal. Nos résultats montrent par ailleurs que les flux nuls des matrices origine-destination contiennent une information qu'il est utile de ne pas négliger.

MOTS-CLES : DONNÉES AGRÉGÉES, DEMANDE DE TRANSPORT DIRECTE, MOINDRES CARRÉS, TOBIT, MONTRÉAL

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1. INTRODUCTION

Applications of aggregate **spatial** direct demand models to interzonal urban transportation are seldom discussed in the literature. A few attempts to build such models have been made in the past (see for example, Domencich and Kraft (1), Ch. 2, Algers (2)). However, probably because of the difficulties encountered in calibrating the models, despite the good quality of the data available from urban transport surveys, this type of modelization seems to have been abandoned by most researchers.

To illustrate the difficulties encountered in calibrating such models, one can refer to the above cited studies. In their two-equation model for explaining urban interzonal passenger travel demand by automoile and public transit, Domencich and Kraft used travel time and travel cost by the two transportation modes, as well as socioeconomic and land-use characteristics, as explanatory variables. Constrained multiple regression estimation procedures had to be used to ensure that no coefficient had the wrong sign, since attemps of calibration by unconstrained regression techniques yielded **nonsensical results**. Among the problems explaining the need to have recourse to constrained estimation techniques, the authors mention the size inequalities of the zones and the collinearity between some socioeconomic characteristics such as car ownership and income.

Algers reports the results of studies made for explaining interzonal passenger trips by car and public transportation in Stockholm. The same types of explanatory variables were used as in the previous study, but unconstrained regression analysis was used. In all cases, some of the estimated coefficients turned out to have the **wrong sign**. In most cases, these coefficients were associated with the travel time or cost variables or with the car ownership variable. Only one model for explaining total interzonal trips, which incorporated very few variables and

strong prior constraints, yielded satisfactory results. This model had only three explanatory variables : the attraction of the zone of destination, the generation of the zone of origin and a **single composite** variable as a measure of generalized travel cost. The very fact of using a single generalized impedance which encompasses time and money costs associated with both the automobile and the public transport system, and which also depends on car ownership, is indeed akin to using these separate variables with prior constraints in their coefficients.

In contrast with the approaches discussed above, the purpose of the present paper is to suggest that appropriate functional form selection and adequate parameter interpretation may eliminate the above mentioned paradoxes and produce satisfactory results. The application considered in the following pages is a one-equation model for explaining interzonal work trips during the autumn of 1970 in Montreal. Section 2 presents the basic multiple regression model applied in this study as well as the data. Section 3 suggests alternative specifications that yield more reasonable results. Section 4 discusses the results obtained when the Tobit model for limited dependent variables is used. In conclusion, further investigation of a generalized version of the model provisionally retained is suggested.

2. THE MODEL AND THE DATA

The purpose of our model is to "explain" work trips between zones, at morning peak hours, in Montreal. For that purpose, the Montreal region has been divided into 26 zones. The explanatory variables considered include network variables, namely travel costs and travel time, and socio-economic characteristics of the zones and their population. In Montreal, the public transit fare is fixed over the whole territory served by the Transport Commission of the Montreal Urban Community. Travel cost by car

is essentially a function of distance and parking cost at destination. Travel time may also influence travel cost by car, but this influence is likely to be marginal. Distance and travel time by car are among the variables included in the model. Since no complete information was available on parking costs, dummy variables were used to distinguish between more commercial and more residential destination zones as well as between destination zones located in the center of the city and those located on the outskirts, because parking costs are likely to vary according to these zonal characteristics. Clearly, these dummy variables may embody simultaneously numerous other effects associated with the location of the zones considered. For that reason, similar dummies were also used for the zones of origin.

More specifically, our model may be expressed essentially as follows :

$T_{ij} = f[(D_{ij}, TC_{ij}, TT_{ij}), (L_i, J_j), (Y_i, A_i), (CS_i, RS_i, CD_i, RD_i, CS_j RS_j, CD_j, RD_j)]$, where

T_{ij}, the **dependent variable** , is the number of trips from zone i to zone j, either by car or by public transit, namely bus and metro. (Other means of transportation such as riding or walking were not considered.) The data are obtained from a 4% sample of the households;

D_{ij}, TC_{ij}, TT_{ij} **are network variables** :

- D_{ij} = distance between i and j;
- TC_{ij} = total travel time by car between i and j. The data are obtained from a shortest path assignment using observed link travel times;
- TT_{ij} = total travel time by public transit between i and j. The data are obtained from Transcom (3) a shortest path transit assignment procedure which takes wait-time, transfer time and in-vehicle time into account : these elements are not weighted to best reproduce link flows;

L_i, J_j are **activity variables** :

L_i = labor force residing in zone of origin i;

J_j = jobs located in zone of destination j;

Y_i, A_i are **socioeconomic variables** :

Y_i = personal income in zone i;

A_i = number of automobiles owned by residents of zone i;

CS_k, RS_k, CD_k, RC_k, for k = i,j, are **dummy variables** :

CS_k = 1, if zone k is a commercial zone located in the suburbs;
0, otherwise;

RS_k = 1, if zone k is a residential zone located in the suburbs;
0, otherwise;

CD_k = 1, if zone k is a commercial zone located downtown;
0, otherwise;

RC_k = 1, if zone k is a residential zone located downtown;
0, otherwise;

and f(.) designates a **linear function.**

Since we excluded intrazonal trips because of the difficulty of defining their travel time, and zone 26 is a residual zone, the total number of available observations was 600. For more than half the observations, the survey on which our data are based reported zero trips[1]. Following the traditional practice, the first results reported below are based on the subset of 249 observations with $T_{ij} > 0$. However, since the numerous observations with $T_{ij} = 0$ also contain valuable information, we also report results where we included these observations, applying appropriate regression techniques for limited dependent variables. The fact of

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[1] This low percentage is due to the large size of our 25 zones, which on average had a population of 23,200.

including these null observations appears as a significant improvement over the traditional practice, since these observations do contain useful information.

Since the zones considered are of unequal size, one is likely to encounter severe problems of heteroskedasticity if T_{ij} is used as such, as the dependent variable. There are several possible ways of reducing the problems of heteroskedasticity. A usual procedure is to adopt a per capita (or similar standardization) formulation. Therefore, a more adequate formulation of our model appears to be the following :

$$TL_{ij} = \beta_0 + (\beta_1 D_{ij} + \beta_2 TC_{ij} + \beta_3 TT_{ij}) + (\beta_4 LJ_i + \beta_5 JL_j) + (\beta_6 YL_i + \beta_7 AL_i)$$
$$+ (\beta_8 CS_i + \beta_9 CD_i + \beta_{10} RD_i + \beta_{11} CS_j + \beta_{12} RS_j + \beta_{13} RD_j) + u_{ij}$$

where

$TL_{ij} = T_{ij}/L_i$, $LJ_i = L_i/J_i$, $JL_j = J_j/L_j$, $YL_i = Y_i/L_i$ and $AL_i = A_i/L_i$.

Ideally, it would have been preferable still to deflate the number of trips by the number of employed people residing in the zone of origin rather than by the labor force, since unemployed people do not make work trips. Unfortunately, the breakdown of the labor force residing in any zone between the employed and the unemployed, was not available.

Note that, as usual, one dummy variable relating to the zones of origin and one dummy variable relating to the zones of destination had to be removed to avoid perfect collinearity. The dummies removed are RS_i and CD_j. The coefficients of the remaining dummies must therefore be interpreted as measuring differential effects.

In the above formulation, the former dependent variable T_{ij} has been standardized by dividing it by the number of persons in the labor force, in the zone of origin. It could alternatively have been divided by

the number of jobs available in the zone of destination of by some combination of L_i and J_j.

The coefficients of equation (1) were estimated by maximizing the log-likelihood of the T_{ij}, assuming the residual errors to be normally and independently distributed $N(0,\sigma^2)$. This log-likelihood has the following form :

$$L = -\frac{N}{2} \ln (2\pi) + \sum_{\ell=1}^{N} \ln J_\ell - N \ln \sigma - \sum_{\ell=1}^{N} (Y_\ell - X_\ell \beta)^2 / 2\sigma^2 ,$$

where ℓ designates the observation number, Y_ℓ corresponds to the LHS of equation (1) for a given observation ℓ, $X_\ell\beta$ corresponds to the nonramdom part of the RHS, and J_ℓ is the jacobian of the transformation from TL_{ij} to T_{ij}. For any given observation with origin i and destination j, this jacobian is equal to $1/L_i$. Note that this jacobian plays no role here in the maximization of L since TL_{ij} is not a function of any unknown parameter. It would be different if TL_{ij} did involve unknown parameters as in the model suggested in the conclusion.

The results are shown in column 1R of Table 1. These results show the elasticity associated with each variable, evaluated at the sample means taking only positive values into account[2], and also the t-statistics associated with the regression coefficients.

These results present one major anomaly. The elasticity associated with the **distance** variable is positive[3]. One would expect distance to have a significant negative effect on work trip demand.

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[2] This is also true of dummy variables and constitutes an appropriate evaluation of their influence on the dependent variables, as shown in Section 5 of Dagenais et al. (4). In Table 1, the code names of dummy variables are underlined twice.

[3] That result also occurs if T_{ij} is used at the dependent variable in Eq. (1) instead of TL_{ij}; the alternative specification used in the next section also corrects the problem in that case.

3. ALTERNATIVE SPECIFICATIONS

This paradoxical result comes from the fact that two other variables included in the regression model as **network variables** also incorporate a distance effect, namely travel time by car and by public transit. A neater interpretation of the results may be obtained if one considers that, for a given distance, the average traveling time involved corresponds to the normal speed by car and public transit in the city. If however, for a given distance, the average traveling speed between two zones, by car or by public transit, is greater than this normal speed, this will have a positive effect on trip demand between these zones.

These considerations suggest that better specification of our regression model might be achieved by replacing the triplet $(D_{ij}, TC_{ij} TT_{ij})$ by $(D_{ij}, AS_{ij}, TS_{ij})$, where AS_{ij} denotes the travel **speed by car** and TS_{ij} denotes the travel **speed by public transit** between zone i and zone j.

Our model becomes :

$$(2)\quad TL_{ij} = \beta_0 + (\beta_1 D_{ij} + \beta_2 AS_{ij} + \beta_3 TS_{ij}) + (\beta_4 LJ_i + \beta_5 JL_j) + (\beta_6 YL_i + \beta_7 AL_i) + (\beta_8 CS_i + \beta_9 CD_i + \beta_{10} RD_i + \beta_{11} CS_j + \beta_{12} RS_j + \beta_{13} RD_j) + u_{ij}.$$

The demand elasticities of equation (2) are shown in column 2 of Table 1. The elasticity associated with D_{ij} now has the expected negative sign and its t-statistic is relatively large. The variables AS_{ij} and TS_{ij} have positive elasticities as anticipated[4]. The work trip elasticity associated with TS_{ij}, public transit traveling **speed** is relatively small however and has a low t-statistic.

But the **socioeconomic variables** appearing in equation (2) have very low t-statistics, thus suggesting that one could not reject the

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[4] Longer distances decrease the time budget available to consumers; speed variables function as inverse time prices.

TABLE 1

Elasticities and t-statistics of regression coefficients

	CODE NO.	1R	2R	3R	1T	2T	3T
	DEP.VAR.	TIJ/LFTO	TIJ/LFTO	TIJ/LFJB	TIJ/LFTO	TIJ/LFTO	TIJ/LFJB
P = TIMES AND PRICES							
TRAVEL TIME BY CAR	TTAU	-1.260			-.811		
		(-2.25)			(-2.56)		
TOTAL TRAVEL TIME BY TRANSIT	TOTTR	-.363			-.630		
		(-.57)			(-2.24)		
DISTANCE BY CAR	DISTAU	.746	-.801	-1.130	.132	-1.130	-1.420
		(1.20)	(-2.92)	(-5.67)	(.44)	(-6.66)	(-8.28)
SPEED BY CAR (DISTAU/TTAU)	VAU		1.180	.934		.622	.507
			(2.52)	(2.31)		(2.03)	(1.68)
SPEED BY TRANSIT (DISTAU/TOTTR)	VTR		.261	.324		.394	.518
			(.48)	(.79)		(1.29)	(1.71)
A = ACTIVITIES							
LABOR FORCE PER JOB AT ORIGIN	LFTO/JOB	.157	.170	.011	.090	.099	.016
		(1.72)	(1.83)	(.10)	(1.10)	(1.20)	(.20)
JOBS PER WORKER AT DEST.	JOBD/LFT	.616	.517	.146	1.660	1.610	1.470
		(1.99)	(1.54)	(.61)	(7.93)	(7.59)	(7.00)
V = SOCIOECONOMIC							
INCOME PER WORKER AT ORIGIN	VPW	-.107	-.077	-.716	-.622	-.601	-1.080
		(-.32)	(-.23)	(-1.91)	(-2.16)	(-2.07)	(-3.73)
CARS PER WORKER	CARPW	.555	.691	2.070	1.150	1.170	2.220
		(.49)	(.57)	(1.93)	(1.78)	(1.78)	(3.39)
E = ENVIRONMENT							
DOWNTOWN AND COMMERCIAL AT ORIGIN	CVC-OR	.250	.346	.697	.268	.298	.578
	******	(.24)	(.30)	(.78)	(.56)	(.62)	(1.20)
DOWNTOWN AND RESIDENTIAL AT ORIGIN	CVR-OR	.318	.355	.436	.319	.325	.394
	******	(1.06)	(1.20)	(1.68)	(1.67)	(1.88)	(2.30)
DOWNTOWN AND RESIDENTIAL AT DEST.	CVR-DE	-1.974	-2.445	-.770	3.167	2.861	5.583
	******	(-1.27)	(-1.46)	(-.63)	(2.78)	(2.51)	(4.91)
SUBURBIAN AND COMMERCIAL AT ORIGIN	BC-OR	.232	.224	-.281	-.486	-.511	-.788
	*****	(.75)	(.74)	(-.99)	(-2.33)	(-2.44)	(-3.76)
SUBURBIAN AND COMMERCIAL AT DEST.	BC-DE	-2.504	-2.912	-.674	1.644	1.369	4.356
	*****	(-1.80)	(-2.01)	(-.66)	(1.72)	(1.42)	(4.57)
SUBURBIAN AND RESIDENTIAL AT DEST.	BR-DE	-1.967	-2.460	-.660	3.068	2.773	5.523
	*****	(-1.24)	(-1.45)	(-.53)	(2.72)	(2.43)	(4.89)
REGRESSION CONSTANT	CONSTANT	1.090	-.095	-2.310	-3.330	-4.260	-6.960
		(.56)	(-.05)	(-1.46)	(-2.60)	(-3.21)	(-5.26)
LOG-LIKELIHOOD		-2529.451	-2529.954	-2385.166	-989.193	-992.294	-889.578
PSEUDO-R2		.729	.728	.385	.773	.770	.625
NUMBER OF OBSERVATIONS		249	249	249	600	600	600
ESTIMATION PERIOD		1-249	1-249	1-249	1-600	1-600	1-600
NUMBER OF INDEPENDENT VARIABLES		14	14	14	14	14	14

hypothesis that these variables have no effect on work trip demand. This result is somewhat unexpected and constitutes an incitation to investigate alternative functional forms that could yield sharper results for the coefficients of these variables.

As mentioned earlier, other possible functional forms could be obtained by modifying the dependent variable. For example, TL_{ij} could be replaced by TLJ_{ij} where

$$TLJ_{ij} = \frac{T_{ij}}{(L_i J_j)^{\frac{1}{2}}} . \tag{3}$$

Column 3R of Table 1 gives the results of this new specification. Note that the log-likelihood function for model 3R is notably higher than that for model 2R. Since these two models can be seen as nested alternatives of a more general model, where $TLJ_{ij} = T_{ij}/(L_i^{\alpha_1} J_j^{\alpha_2})$ with $\alpha_1 = 1$, $\alpha_o = 0$ in model 1 and $\alpha_1 = \alpha_2 = 1/2$ in model 2, our results suggest that model 3 ought to be preferred to model 2.

It appears now that, in the latter model, the two socioeconomic variables YL_i and AL_i have relatively high t-statistics. The elasticity of AL_i is positive as anticipated, but the elasticity of YL_i is negative, while one might have expected it to be positive. One of the reasons why one might anticipate that zones with higher income per worker would generate more work trips is that these zones would also have a greater number of cars per worker, and car is the main mode of transportation to work. However, one must note that our model also includes a car variable AL_i and hence, what our results imply is that, everything else being equal including the number of cars per worker, a zone with a higher income per worker will generate fewer work trips towards the other zones. This may simply suggest that travel to work is an inferior good. Those who can afford it tend to reduce the distance between their residence and their work place.

Indeed, the survey from which our data are drawn indicates that, in Montréal, 30% of the people living downtown walk to work. Therefore, it is not all implausible to presume that richer people tend to choose their residence and their work place in such a way as to reduce the distance between these two poles.

The elasticities and t-statistics of the **activity variables** drop considerably when the formulation of model 3R is used. One might argue that because the effects of labor force availability at the origin and job availability at the destination are already taken account of in the denominator of the new dependent variable, the explanatory variables LJ_i and JL_j could now be dropped altogether from the R.H.S. of the regression model. This is not quite true, however, since LJ_i and JL_j include respectively J_i and L_j in their denominators and these variables are not included in TLS_{ij}. The results of column 3R rather suggest that further fine tuning might be required in assigning the exponential coefficients to L_i and J_j in the denominator of TLS_{ij}, as will be further discussed in the concluding section.

It is to be noted also that, with the formulation of model 3R, the t-statistics of the **dummy variables** are quite low, which suggests that some of these dummies could be eliminated.

Before proceeding to any further refinement of this latter specification, it was felt advisable to verify whether taking account of the information contained in the null observations would suggest the same course of action.

4. TAKING ACCOUNT OF THE NULL OBSERVATIONS

The Tobit model for dependent variables with a zero lower limit was applied to our complete set of 600 observations, which includes 351 null observations. The same three specifications were used as with the multiple regression model. The results are given in columns 1T to 3T. The elasticities are evaluated at the same points and in the same manner as for the corresponding regression models, to facilitate comparisons[5]. The socio-economic variables have now relatively high asymptotic t-statistics in all three models and the income elasticity retains its negative sign. The JL_j variable is also always highly significant while the LJ_i variable is not and its t-statistics is particularly low in column 3T. There seems to be an asymmetry in the way labor force self sufficiency at the origin and job self sufficiency at the destination determine demand flows.

Here again, as previously with the multiple regression models, the elasticity of the D_{ij} variable has the wrong positive sign in 1T and the expected negative sign in 2T and 3T. The log-likelihood function increases again notably when one moves from model 2T to model 3T, thus suggesting that model 3T is also to be preferred. The nonnegligible changes observed in some of the elasticities and t-statistics between model 2T and 3T suggest again that further investigation of the appropriate exponential coefficients to assign to the L_i and J_j variables in the denominator of TLS_{ij} might be warranted.

We also note that the dummy variables are very significant in columns 1T to 3T and that half of them have changed signs. The addition of information gained by using the null observations of the origin-desti-

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[5] For the Tobit model, elasticities could alternatively have been measured as proportional changes in the **expected** value of the dependent variable due to proportional changes in the different explanatory variables. If the expected value of the dependent variable is sufficiently far above the zero limit, this alternative elasticity measure will be very similar to that adopted here.

nation matrix implies that the elimination of dummies evoked in our comment about column 3R would have been a mistake.

5. CONCLUSION

This paper constitutes only a progress report. Further modifications of the dependent variable could be explored. Instead of deciding a priori that the dependent variable should have the form T_{ij}/L_i or $T_{ij}/(L_iJ_j)^{\frac{1}{2}}$, it would be preferable to adopt a more general form such as $T_{ij}/(L_i^{\alpha_1}J_j^{\alpha_2})$ and estimate the α's from the data. The regression residuals should also be carefully examined to investigate other possible problems of mispecification which might not yet have been detected at this stage.

The results obtained so far, however, are sufficiently coherent to suggest that aggregate direct travel demand models can work. They also show that taking the zero observations of origin-destination matrices into account, instead of neglecting them, greatly increases the efficiency of the results.

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